

Can $N^*(1535)$ be a probe to observe the partial restoration of chiral symmetry in nuclear matter?

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(Dated: April 13, 2017)

We investigate modifications of mass and decay width of $N^*(1535)$ in nuclear matter. The nucleon and $N^*(1535)$ is introduced by a parity doublet model, and nuclear matter is constructed by one loop of the nucleon and $N^*(1535)$. The decay width of $N^*(1535)$ is studied with respecting the gap equation. Our calculations show that the partial width of $\Gamma_{N^* \rightarrow N\pi}$ is slightly broadened by the collisional broadening, and that of $\Gamma_{N^* \rightarrow N\eta}$ is drastically suppressed at density. As a result, the total decay width Γ_{tot} gets small at density. These modifications, especially the drastic narrowing of partial width of $\Gamma_{N^* \rightarrow N\eta}$, together with the dropping of mass of $N^*(1535)$ provide experiments for observing the partial restoration of chiral symmetry in nuclear matter by means of $N^*(1535)$ resonance with useful information.

I. INTRODUCTION

Investigating chiral symmetry is one of the most important subject in Quantum Chromodynamics (QCD), since hadron masses can be explained by the spontaneous breakdown of chiral symmetry. Although chiral symmetry is broken in the vacuum, it is expected to be restored at temperature and/or density, so that the search for chiral symmetry at such extreme environments is receiving attention recently.

One of a powerful tool to investigate the relation between hadron properties and chiral symmetry is a hadron effective model. One example is the linear sigma model which was introduced by Gell-Mann and Levy [1, 2]. In this model, the linear representation of $SU(2)_L \times SU(2)_R$ group is employed for the nucleon, and mass generation of the nucleon is demonstrated. The linear representation was extended to the parity doublet model [3]. In this model, a positive parity nucleon and an excite negative parity nucleon, such as the nucleon and $N^*(1535)$, can be studied collectively by employing a mirror assignment. Under this assignment, the negative parity nucleon is regarded as the chiral partner to the positive parity nucleon, so that these gets degenerated when the chiral symmetry restoration occurs. This idea was further extended to other excited states and delta isobars [4–7].

Some theoretical studies based on hadron effective models show a tendency of chiral restoration in nuclear matter [9, 10]. Other chiral effective models such as the Nambu-Jona-Lasinio (NJL) model reads the same tendency (see Ref. [8] for a review and references therein). Also, modifications of hadrons in nuclear matter as probes to understand the partial restoration of chiral symmetry were also studied [11, 12]. However, it is not an easy work to confirm it experimentally (see Ref. [13] for a review and references therein). Investigating the chiral symmetry in nuclear matter by focusing on modifications of charmed mesons were done as well [14–16].

In order to see an indication of chiral restoration in nuclear matter, it is worth focusing on modifications of properties of $N^*(1535)$ in nuclear matter. As is well known, the observed branching ratio of $N^* \rightarrow N\eta$ mode is $\Gamma_{N^* \rightarrow N\eta}/\Gamma_{\text{tot}} \sim 50\%$ in the vacuum while threshold of $N + \eta$ is closed to the mass of $N^*(1535)$, which implies $N^*(1535)$ is strongly coupled to η meson and the nucleon. Hence, the partial restoration of chiral symmetry in nuclear matter can have a significant influence on the decay properties of $N^* \rightarrow N\eta$, since mass difference between $N^*(1535)$ and the nucleon gets small as chiral symmetry is restored, by regarding $N^*(1535)$ as a chiral partner to the nucleon within the parity doublet model. Studies on $N^*(1535)$ paying attention to chiral symmetry in-medium exist [26–28].

Studies on $N^*(1535)$ is important in the context of η mesic nuclei which was first reported by Haider and Liu [17] as well, since $N^*(1535)$ is strongly coupled to ηN system as we have already stated, and it is expected that $N^*(1535)$ resonance plays a significant role to form a η -nucleus bound state. The quest for such exotic nucleus has been animatedly performed theoretically [18–22] and experimentally [23–25]. Such studies can also leads to understanding of the partial restoration of chiral symmetry in nuclear matter in laboratories.

In Ref. [29], a parity doublet model in which ω meson and ρ meson contributions as well as σ meson and pion are included was considered, and the properties of nuclear matter, i.e., the nuclear saturation density, the binding energy, the incompressibility and the symmetry energy, can be reproduced within this model at mean field level. In this paper, we extended the parity doublet model in Ref. [29] by taking fluctuations, and calculate modifications of mass and decay width of $N^*(1535)$ in nuclear matter, to provide useful information to observe the partial restoration of chiral symmetry in nuclear matter.

This paper is organized as following. In Sec. II, we construct the parity doublet model which were proposed in Ref. [29], and determine model parameters. Derivative couplings and $NN^*\eta$ couplings are also included to explain the decay properties of $N^*(1535)$ in the vacuum. In

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Sec. III, we solve a gap equation respect to mean field of σ meson in nuclear matter, and mass modifications of the nucleon and $N^*(1535)$ are studied. Fluctuations of pion and η meson are derived as well. In Sec. IV, we demonstrate a way to calculate the decay width of $N^*(1535)$ in nuclear matter with respecting the gap equation, and give results. In Sec. V, we summarize the present study and give some discussions.

II. MODEL CONSTRUCTION

In the present study, we shall investigate modifications of decay properties of $N^*(1535)$ in nuclear matter. In order to treat $N^*(1535)$ and the nucleon collectively, we construct an effective model for $N^*(1535)$ and the nucleon within the parity doublet model [3] in this section. In this model, a nucleon which carries positive parity is regarded as the chiral partner of a nucleon which carries negative parity, so that the masses of them get degenerated when the chiral symmetry is restored. Here, we regard the nucleon as the positive parity nucleon while $N^*(1535)$ as the negative parity nucleon.

The nucleon and $N^*(1535)$ are introduced via two Fermion fields ψ_1 and ψ_2 which transform under the $SU(2)_L \times SU(2)_R$ chiral transformation as

$$\begin{aligned}\psi_{1l} &\rightarrow g_L \psi_{1l}, & \psi_{1r} &\rightarrow g_R \psi_{1r}, \\ \psi_{2l} &\rightarrow g_R \psi_{2l}, & \psi_{2r} &\rightarrow g_L \psi_{2r}.\end{aligned}\quad (1)$$

ψ_{1l} , ψ_{1r} , ψ_{2l} and ψ_{2r} are defined by

$$\begin{aligned}\psi_{1l(2l)} &= \frac{1 - \gamma_5}{2} \psi_{1(2)}, \\ \psi_{1r(2r)} &= \frac{1 + \gamma_5}{2} \psi_{1(2)},\end{aligned}\quad (2)$$

and g_L and g_R are elements of $SU(2)_L$ and $SU(2)_R$, respectively. To start with, we need to construct a Lagrangian which is invariant under the $SU(2)_L \times SU(2)_R$ chiral symmetry, parity and charge conjugation. By introducing a chiral field M including σ meson and pion which transforms under the $SU(2)_L \times SU(2)_R$ chiral transformation as

$$M \rightarrow g_L M g_R^\dagger, \quad (3)$$

Lagrangian up to $O(\partial M^2)$ is given by

$$\begin{aligned}\mathcal{L}_N &= \bar{\psi}_{1r}(i\partial\!\!\!/ + \gamma_0\mu_B)\psi_{1r} + \bar{\psi}_{1l}(i\partial\!\!\!/ + \gamma_0\mu_B)\psi_{1l} \\ &+ \bar{\psi}_{2r}(i\partial\!\!\!/ + \gamma_0\mu_B)\psi_{2r} + \bar{\psi}_{2l}(i\partial\!\!\!/ + \gamma_0\mu_B)\psi_{2l} \\ &- m_0 [\bar{\psi}_{1l}\psi_{2r} - \bar{\psi}_{1r}\psi_{2l} - \bar{\psi}_{2l}\psi_{1r} + \bar{\psi}_{2r}\psi_{1l}] \\ &- g_1 [\bar{\psi}_{1r}M^\dagger\psi_{1l} + \bar{\psi}_{1l}M\psi_{1r}] - g_2 [\bar{\psi}_{2r}M\psi_{2l} + \bar{\psi}_{2l}M^\dagger\psi_{2r}] \\ &- ih_1 [\bar{\psi}_{1l}(M\partial\!\!\!/M^\dagger - \partial\!\!\!/MM^\dagger)\psi_{1l} + \bar{\psi}_{1r}(M^\dagger\partial\!\!\!/M - \partial\!\!\!/M^\dagger M)\psi_{1r}] \\ &- ih_2 [\bar{\psi}_{2r}(M\partial\!\!\!/M^\dagger - \partial\!\!\!/MM^\dagger)\psi_{2r} + \bar{\psi}_{2l}(M^\dagger\partial\!\!\!/M - \partial\!\!\!/M^\dagger M)\psi_{2l}].\end{aligned}\quad (4)$$

m_0 , g_1, g_2 , h_1 and h_2 in Eq. (4) are real parameters which will be determined later, and μ_B is a baryon number chemical potential added to access to finite density. Note that derivative couplings are included as will be explained later.

The Lagrangian for σ meson and pion can be given by

$$\begin{aligned}\mathcal{L}_M &= \frac{1}{4}\text{Tr}[\partial_\mu M \partial^\mu M^\dagger] + \frac{\bar{\mu}^2}{4}\text{Tr}[MM^\dagger] - \frac{\lambda}{16}(\text{Tr}[MM^\dagger])^2 \\ &+ \frac{\lambda_6}{48}(\text{Tr}[MM^\dagger])^2 + \frac{\epsilon}{4}(\text{Tr}[\mathcal{M}^\dagger M] + \text{Tr}[M^\dagger \mathcal{M}]),\end{aligned}\quad (5)$$

where $\bar{\mu}$, λ , λ_6 , ϵ and real parameters. The last term in the second line in Eq. (5) which explicitly breaks the

chiral symmetry is added to reproduce the finite mass of pion. \mathcal{M} represents a current quark mass matrix which has the form

$$\mathcal{M} = \begin{pmatrix} \bar{m} & 0 \\ 0 & \bar{m} \end{pmatrix} \quad (6)$$

when the isospin symmetry is assumed.

In the present analysis, we follow Ref. [29] to determine model parameters. In this reference, Lagrangians (4) and (5) were extended to include ρ meson by using the polar decomposition $M = \sigma U$ with $U = \exp(i\pi^a \tau^a / f_\pi)$ (τ^a is the Pauli matrix and a runs over $a = 1, 2, 3$) via the Hidden Local Symmetry (HLS) [30]. Then we rewrite Lagrangians (4) and (5) as

$$\begin{aligned}\mathcal{L}_N &= \bar{\psi}_{1r}(i\partial\!\!\!/ + \gamma_0\mu_B)\psi_{1r} + \bar{\psi}_{1l}(i\partial\!\!\!/ + \gamma_0\mu_B)\psi_{1l} + \bar{\psi}_{2r}(i\partial\!\!\!/ + \gamma_0\mu_B)\psi_{2r} + \bar{\psi}_{2l}(i\partial\!\!\!/ + \gamma_0\mu_B)\psi_{2l} \\ &- m_0 [\bar{\psi}_{1l}\psi_{2r} - \bar{\psi}_{1r}\psi_{2l} - \bar{\psi}_{2l}\psi_{1r} + \bar{\psi}_{2r}\psi_{1l}] - g_1\sigma_0 [\bar{\psi}_{1r}\psi_{1l} + \bar{\psi}_{1l}\psi_{1r}] - g_2\sigma_0 [\bar{\psi}_{2r}\psi_{2l} + \bar{\psi}_{2l}\psi_{2r}] \\ &- g_1 \left[\bar{\psi}_{1r} \left(\sigma - i \frac{\sigma_0}{f_\pi} \pi \right) \psi_{1l} + \bar{\psi}_{1l} \left(\sigma + i \frac{\sigma_0}{f_\pi} \pi \right) \psi_{1r} \right] - g_2 \left[\bar{\psi}_{2r} \left(\sigma + i \frac{\sigma_0}{f_\pi} \pi \right) \psi_{2l} + \bar{\psi}_{2l} \left(\sigma - i \frac{\sigma_0}{f_\pi} \pi \right) \psi_{2r} \right]\end{aligned}$$

$$\begin{aligned}
& + \frac{g_1}{2\sigma_0} \frac{\sigma_0^2}{f_\pi^2} [\bar{\psi}_{1r}\pi^2\psi_{1l} + \bar{\psi}_{1l}\pi^2\psi_{1r}] + \frac{g_2}{2\sigma_0} \frac{\sigma_0^2}{f_\pi^2} [\bar{\psi}_{2r}\pi^2\psi_{2l} + \bar{\psi}_{2l}\pi^2\psi_{2r}] \\
& + h_1 \left[-\frac{2\sigma_0^2}{f_\pi} \bar{\psi}_{1l}\not{\partial}\pi\psi_{1l} + \frac{2\sigma_0^2}{f_\pi} \bar{\psi}_{1r}\not{\partial}\pi\psi_{1r} - i\frac{\sigma_0^2}{f_\pi^2} \bar{\psi}_{1l}[\pi, \not{\partial}\pi]\psi_{1l} - i\frac{\sigma_0^2}{f_\pi^2} \bar{\psi}_{1r}[\pi, \not{\partial}\pi]\psi_{1r} \right] \\
& + h_2 \left[-\frac{2\sigma_0^2}{f_\pi} \bar{\psi}_{2r}\not{\partial}\pi\psi_{2r} + \frac{2\sigma_0^2}{f_\pi} \bar{\psi}_{2l}\not{\partial}\pi\psi_{2l} - i\frac{\sigma_0^2}{f_\pi^2} \bar{\psi}_{2r}[\pi, \not{\partial}\pi]\psi_{2r} - i\frac{\sigma_0^2}{f_\pi^2} \bar{\psi}_{2l}[\pi, \not{\partial}\pi]\psi_{2l} \right] \\
& + \dots,
\end{aligned} \tag{7}$$

and

$$\begin{aligned}
\mathcal{L}_M = & \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma + \frac{\sigma_0^2}{2f_\pi^2} \partial_\mu \pi^a \partial^\mu \pi^a - \frac{\lambda}{4} (\sigma_0 + \sigma)^2 + \frac{\lambda_6}{6} (\sigma_0 + \sigma)^6 + \bar{m} \epsilon \sigma_0 - \frac{\epsilon \bar{m}}{2} \frac{\sigma_0}{f_\pi^2} \pi^a \pi^a \\
& + \dots,
\end{aligned} \tag{8}$$

respectively. In obtaining Eqs. (7) and (8), we have assumed spatially homogeneous spontaneous breakdown of chiral symmetry, so that M is replaced as $M = \sigma U \rightarrow (\sigma_0 + \sigma)U$ (σ_0 is the constant mean field of σ meson, and σ represents a fluctuation of scalar mode on such background). The ellipsis in Eqs. (7) and (8) includes higher order of interactions. In order to define the mass eigenstate of positive parity nucleon N_+ and negative parity nucleon N_- , we need to diagonalize the mass matrix in

Lagrangian (7) by introducing a mixing angle θ :

$$\begin{pmatrix} N_+ \\ N_- \end{pmatrix} = \begin{pmatrix} \cos \theta & \gamma_5 \sin \theta \\ -\gamma_5 \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}. \tag{9}$$

As already mentioned, N_+ is regarded as the nucleon while N_- is regarded as $N^*(1535)$. Hence, using Eqs. (2) and (9), Lagrangians (7) and (8) yield

$$\begin{aligned}
\mathcal{L}_N = & \bar{N}_+ (i\not{\partial} + \gamma_0 \mu_B) N_+ + \bar{N}_- (i\not{\partial} + \gamma_0 \mu_B) N_- - m_+ \bar{N}_+ N_+ - m_- \bar{N}_- N_- \\
& - g_{NN\sigma} \bar{N}_+ \sigma N_+ - g_{NN\pi} \bar{N}_+ i\gamma_5 \pi_r N_+ + g_{NN^*\sigma} \bar{N}_+ \gamma_5 \sigma N_- + g_{NN^*\pi} \bar{N}_+ i\pi_r N_- \\
& - g_{NN^*\sigma} \bar{N}_- \gamma_5 \sigma N_+ - g_{NN^*\pi} \bar{N}_- i\pi_r N_+ - g_{N^*N^*\sigma} \bar{N}_- \sigma N_- - g_{N^*N^*\pi} \bar{N}_- i\gamma_5 \pi_r N_- \\
& + \frac{g_{NN\sigma}}{2\sigma_0} \bar{N}_+ \pi_r^2 N_+ - \frac{g_{NN^*\sigma}}{2\sigma_0} \bar{N}_+ \gamma_5 \pi_r^2 N_- + \frac{g_{NN^*\sigma}}{2\sigma_0} \bar{N}_- \gamma_5 \pi_r^2 N_+ + \frac{g_{N^*N^*\sigma}}{2\sigma_0} \bar{N}_- \pi_r^2 N_- \\
& + 2\sigma_0 h_{NN\pi} \bar{N}_+ \not{\partial} \pi_r \gamma_5 N_+ + 2\sigma_0 h_{NN^*\pi} \bar{N}_+ \not{\partial} \pi_r N_- + 2\sigma_0 h_{NN^*\pi} \bar{N}_- \not{\partial} \pi_r N_+ + 2\sigma_0 \bar{N}_- \not{\partial} \pi_r \gamma_5 N_- \\
& - i f_\pi h_{NN\pi\pi} \bar{N}_+ [\pi_r, \not{\partial} \pi_r] N_+ - i f_\pi h_{NN^*\pi\pi} \bar{N}_+ [\pi_r, \not{\partial} \pi_r] \gamma_5 N_- \\
& - i f_\pi h_{NN^*\pi\pi} \bar{N}_- [\pi_r, \not{\partial} \pi_r] \gamma_5 N_+ - i f_\pi h_{N^*N^*\pi\pi} \bar{N}_- [\pi_r, \not{\partial} \pi_r] N_- \\
& + \dots,
\end{aligned} \tag{10}$$

and

$$\mathcal{L}_M = \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma + \frac{1}{2} \partial_\mu \pi_r^a \partial^\mu \pi_r^a + \frac{1}{2} \bar{\mu}^2 (\sigma_0 + \sigma)^2 - \frac{1}{4} \lambda (\sigma_0 + \sigma)^4 + \frac{1}{6} \lambda_6 (\sigma_0 + \sigma)^6 + \bar{m} \epsilon \sigma_0 - \frac{1}{2} m_\pi^2 \pi_r^a \pi_r^a + \dots. \tag{11}$$

The coupling constants in Eq. (10) are expressed by mixing angle θ and original ones: g_1 , g_2 , h_1 and h_2 as

$$\begin{aligned}
g_{NN\sigma} &= g_1 \cos^2 \theta - g_2 \sin^2 \theta \\
g_{NN\pi} &= g_1 \cos^2 \theta + g_2 \sin^2 \theta \\
g_{NN^*\sigma} &= g_1 \sin \theta \cos \theta + g_2 \sin \theta \cos \theta \\
g_{NN^*\pi} &= g_1 \sin \theta \cos \theta - g_2 \sin \theta \cos \theta \\
g_{N^*N^*\sigma} &= -g_1 \sin^2 \theta + g_2 \cos^2 \theta \\
g_{N^*N^*\pi} &= -g_1 \sin^2 \theta - g_2 \cos^2 \theta \\
h_{NN\pi} &= h_1 \cos^2 \theta - h_2 \sin^2 \theta
\end{aligned}$$

$$\begin{aligned}
h_{NN^*\pi} &= -(h_1 \sin \theta \cos \theta + h_2 \sin \theta \cos \theta) \\
h_{N^*N^*\pi} &= h_1 \sin^2 \theta - h_2 \cos^2 \theta \\
h_{NN\pi\pi} &= h_1 \cos^2 \theta + h_2 \sin^2 \theta \\
h_{NN^*\pi\pi} &= -(h_1 \sin \theta \cos \theta - h_2 \sin \theta \cos \theta) \\
h_{N^*N^*\pi\pi} &= h_1 \sin^2 \theta + h_2 \cos^2 \theta,
\end{aligned} \tag{12}$$

and θ satisfies a relation

$$\tan 2\theta = \frac{2m_0}{(g_1 + g_2)\sigma_0}. \tag{13}$$

m_+ and m_- in Eq (10) indicate the masses of N_+ (the

nucleon) and N_- ($N^*(1535)$), respectively, and they are given by

$$m_{\pm} = \frac{1}{2} \left[\sqrt{(g_1 + g_2)^2 \sigma_0^2 + 4m_0^2} \mp (g_2 - g_1) \sigma_0 \right]. \quad (14)$$

The “bare” pion mass m_{π}^2 in Eq (11) is

$$m_{\pi}^2 = \frac{\bar{m}\epsilon}{\sigma_0}. \quad (15)$$

In obtaining Eqs. (10) and (11), we have introduced the renormalized pion field π_r^a by $\pi^a = Z^{1/2} \pi_r^a$ with $Z = f_{\pi}^2 / \sigma_0^2$ in such a way that kinetic term of pion field is normalized properly.

In Ref. [29], only m_0 which is so-called chiral invariant mass is remained to be a free parameter, and the others are fixed. The parameters $g_1, g_2, \bar{m}\epsilon$ are determined by the physical masses of the nucleon, $N^*(1535)$ and pion in the vacuum: $m_+ = 939$ MeV, $m_- = 1535$ MeV and $m_{\pi} = 140$ MeV for a given m_0 . Other parameters $\bar{\mu}^2$, λ and λ_6 are determined by the decay constant in the vacuum $f_{\pi} = 92.4$ [MeV] and properties of nuclear matter (See Ref. [29]. See also Ref. [31] for the detail)¹. The remaining parameters h_1 and h_2 are determined by the decay width of $N^*(1535)$ and axial-charge g_A of the nucleon in the vacuum as following. Although the decay width of $N^*(1535)$ contains a large uncertainty, we assume

$$\Gamma_{N^* \rightarrow N\pi} \sim 75 \text{ MeV}, \quad (16)$$

$$\Gamma_{N^* \rightarrow N\eta} \sim 75 \text{ MeV}, \quad (17)$$

with the total width of $N^*(1535)$ being $\Gamma_{\text{tot}} \sim 150$ MeV [32]. The axial-charge of the nucleon g_A is estimated as $g_A = 1.27$ [32].

From Lagrangian (10), the decay width of $\Gamma_{N^* \rightarrow N\pi}$ is calculated as

$$\Gamma_{N^* \rightarrow N\pi} = \frac{3}{8\pi} \frac{|\vec{q}_{\pi}|}{m_-^2} (F(|\vec{q}_{\pi}|, \Lambda))^2 [(m_+ + m_-)^2 - m_{\pi}^2] \times (g_{NN^*\pi} - 2\sigma_0 h_{NN^*\pi} (m_+ - m_-))^2, \quad (18)$$

where $|\vec{q}_{\pi}|$ is the momentum of emitted pion

$$|\vec{q}_{\pi}| = \frac{\sqrt{[m_-^2 - (m_+ + m_{\pi})^2][m_-^2 - (m_+ - m_{\pi})^2]}}{2m_-^2}. \quad (19)$$

$F(|\vec{q}_{\pi}|, \Lambda)$ in Eq. (18) is the form factor which has the form

$$F(|\vec{q}|, \Lambda) = \frac{\Lambda^2}{|\vec{q}|^2 + \Lambda^2}, \quad (20)$$

¹ In Ref. [29, 31], ω meson and ρ meson contributions are also included, and nuclear saturation density $\rho_B^* = 0.16$ [fm⁻³], binding energy $E/A - m_+ = 16$ [MeV], incompressibility $K = 240$ [MeV] are reproduced. In the present analysis, we do not describe these mesons since we do not adopt fluctuations of them.

and this is inserted in order to take a hadron size. As will be explained later, the value of cutoff parameter Λ is chosen to be $\Lambda = 300$ MeV which is slightly higher than the Fermi momentum of normal nuclear matter density, since we shall study properties of nucleons in nuclear matter in this paper. The axial-charge of the nucleon is taken by introducing an axial gauge field \mathcal{A}_{μ} with the gauge principle in Eq. (10), and reading a coefficient of $\bar{N}_+ \mathcal{A} N_+$ coupling². The resulting coefficient reads

$$g_A = \frac{g_{NN\pi} f_{\pi} + 4f_{\pi}^2 m_+ h_{NN\pi}}{m_+}. \quad (22)$$

Summarizing the above, the model parameters are determined as listed in Tab. I for a given m_0 . The chiral invariant mass m_0 is a free parameter, and we show the values of parameters for $m_0 = 500$ [MeV], $m_0 = 700$ [MeV] and $m_0 = 900$ [MeV] as examples³.

m_0 [MeV]	500	700	900
g_1	9.03	7.82	5.97
g_2	15.5	14.3	12.4
$\hat{\mu}^2$	73.5	30.8	1.74
λ	139	58.8	5.00
$\hat{\lambda}_6$	62.9	25.7	0.952
\hat{h}_1	0.108	0.127	0.145
\hat{h}_2	0.336	0.0473	0.126

TABLE I. Model parameters for a given value of m_0 . The dimensionless parameters $\hat{\mu}^2$, $\hat{\lambda}_6$, \hat{h}_1 and \hat{h}_2 are defined by $\hat{\mu} = \bar{\mu}^2 / f_{\pi}^2$, $\hat{\lambda}_6 = \lambda_6 \cdot f_{\pi}^2$, $\hat{h}_1 = h_1 \cdot f_{\pi}^2$ and $\hat{h}_2 = h_2 \cdot f_{\pi}^2$.

The properties of nuclear matter and decay width of $\Gamma_{N^* \rightarrow N\pi}$ is reproduced by parameter sets in Tab. I. However, we know that $N^*(1535)$ is strongly coupled with η meson as well as pion as indicated in Eq. (17). In order to take into account this large width, we add η mesonic term and an ηNN^* coupling term [18]:

$$\mathcal{L}_{\eta} = \frac{1}{2} \partial_{\mu} \eta \partial^{\mu} \eta - \frac{m_{\eta}^2}{2} \eta^2 + g_{NN^*\eta} \bar{N}^* \eta N + g_{NN^*\eta}^* \bar{N}^* \eta N. \quad (23)$$

The coupling constant $g_{NN^*\eta}$ is determined by the ex-

² Alternatively, g_A can be taken by using the Goldberger-Treiman relation:

$$\frac{G_{\pi NN}}{m_+} = \frac{g_A}{f_{\pi}}, \quad (21)$$

where πNN coupling $G_{\pi NN}$ is obtained as $G_{\pi NN} = g_{NN\pi} + 4f_{\pi} m_+ h_{NN\pi}$ on the on-shell of the nucleon.

³ Eq. (18) is such a quadratic equation respect to $h_{NN^*\pi}$ that we can obtain another solution. This solution reads, however, a relatively larger value of $NN^*\pi\pi$ coupling: $h_{NN^*\pi\pi}$, which is inconsistent with our assumption of Eqs. (16) and (17). Then we have discarded this choice.

perimental value of partial width of $\Gamma_{N^* \rightarrow N\eta}$:

$$\Gamma_{N^* \rightarrow N\eta} = \frac{|g_{NN^*\eta}|^2}{8\pi} \frac{|\vec{q}_\eta|}{m_-^2} (F(|\vec{q}_\eta|, \Lambda))^2 \times [(m_+ + m_-)^2 - m_\pi^2], \quad (24)$$

where $|\vec{q}_\eta|$ is the momentum of emitted η meson

$$|\vec{q}_\eta| = \frac{\sqrt{[m_-^2 - (m_+ + m_\eta)^2][m_-^2 - (m_+ - m_\eta)^2]}}{2m_-^2}. \quad (25)$$

Then Eq. (24) together with Eq. (17) and $m_\eta = 547$ [MeV], we find $|g_{\eta NN^*}| = 2.80$.

In this section, we have obtained the Lagrangian for the nucleon and $N^*(1535)$ interacting with pion and σ meson and η meson which can reproduce the properties of nuclear matter and decay width of $N^*(1535)$. In Sec. III, we show in-medium masses of the nucleon and $N^*(1535)$ and how to adopt the fluctuations of the mesons before studying the modifications of decay width of $N^*(1535)$ in nuclear matter.

III. PARITY DOUBLET MODEL AT DENSITY

A. Gap equation and mass modification of the nucleon and $N^*(1535)$

Here, we construct a nuclear matter by a one loop diagram of the nucleon and $N^*(1535)$, and show the density

dependence of the mean field σ_0 and that of masses of the nucleon and $N^*(1535)$. The effective action by the one loop diagram of N_+ and N_- is provided by performing the path integral as

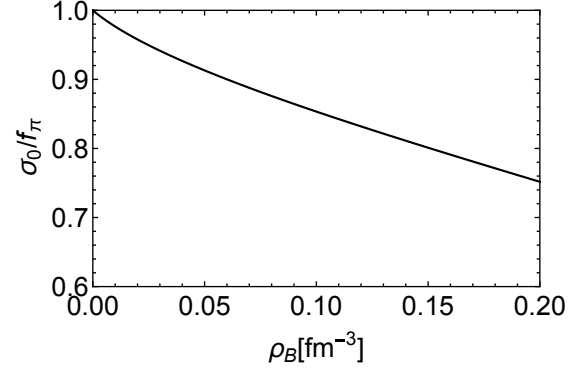


FIG. 1. Density dependence of the mean field σ_0 . As we can see, mean field σ_0 decreases as the density increases, which shows the tendency of partial restoration of chiral symmetry.

$$\Gamma[\sigma_0 + \sigma, \pi_r, \eta] = -i \ln \text{Det} \mathcal{D} + \int d^4x (\mathcal{L}_M + \mathcal{L}_\eta), \quad (26)$$

where matrix \mathcal{D} in Eq. (26) is defined by

$$\mathcal{D} \equiv \begin{pmatrix} \mathcal{D}_{++} & \mathcal{D}_{+-} \\ \mathcal{D}_{-+} & \mathcal{D}_{--} \end{pmatrix},$$

$$\begin{aligned} \mathcal{D}_{++} &= i\cancel{\partial} + \gamma_0 \mu_B - m_+ - g_{NN\sigma}\sigma - g_{NN\pi}i\gamma_5\pi_r + \frac{g_{NN\sigma}}{2\sigma_0}\pi_r^2 + 2\sigma_0\cancel{\partial}\pi_r\gamma_5 - if_\pi h_{NN\pi\pi}[\pi_r, \cancel{\partial}\pi_r], \\ \mathcal{D}_{+-} &= g_{NN^*\sigma}\gamma_5\sigma + g_{NN^*\pi}i\pi_r - \frac{g_{NN^*\sigma}}{2\sigma_0}\gamma_5\pi_r^2 + 2\sigma_0 h_{NN^*\pi}\cancel{\partial}\pi_r - if_\pi h_{NN^*\pi\pi}[\pi_r, \cancel{\partial}\pi_r]\gamma_5, \\ \mathcal{D}_{-+} &= -g_{NN^*\sigma}\gamma_5\sigma - g_{NN^*\pi}i\pi_r + \frac{g_{NN^*\sigma}}{2\sigma_0}\gamma_5\pi_r^2 + 2\sigma_0 h_{NN^*\pi}\cancel{\partial}\pi_r - if_\pi h_{NN^*\pi\pi}[\pi_r, \cancel{\partial}\pi_r]\gamma_5, \\ \mathcal{D}_{--} &= i\cancel{\partial} + \gamma_0 \mu_B - m_- - g_{N^*N^*\sigma}\sigma - g_{N^*N^*\pi}i\gamma_5\pi_r + \frac{g_{N^*N^*\sigma}}{2\sigma_0}\pi_r^2 + 2\sigma_0\cancel{\partial}\pi_r\gamma_5 - if_\pi h_{N^*N^*\pi\pi}[\pi_r, \cancel{\partial}\pi_r]. \end{aligned} \quad (27)$$

“Det” in Eq. (26) stands for the determinant for Dirac, isospin and spacetime indices. When we assume a spacetime-independent mean field σ_0 , the gap equation for σ_0 is given by the stationary point of the effective potential:

$$\frac{\partial V[\sigma_0, 0, 0]}{\partial \sigma_0} = 0 \quad (28)$$

($V[\sigma_0, 0, 0]$ is defined by $\Gamma[\sigma_0, 0, 0] = -V[\sigma_0, 0, 0] \int d^4x$), as

$$\begin{aligned} -\bar{\mu}^2\sigma_0 + \lambda\sigma_0^3 - \lambda_6\sigma_0^5 - \bar{m}\epsilon &= \frac{4m_+^2}{\sqrt{(g_1 + g_2)^2\sigma_0^2 + 4m_0^2}} \int^{k_{F+}} \frac{d^3k}{(2\pi)^3} \frac{1}{2\sqrt{k^2 + m_+^2}} \\ &+ \frac{4m_-^2}{\sqrt{(g_1 + g_2)^2\sigma_0^2 + 4m_0^2}} \int^{k_{F-}} \frac{d^3k}{(2\pi)^3} \frac{1}{2\sqrt{k^2 + m_-^2}}. \end{aligned} \quad (29)$$

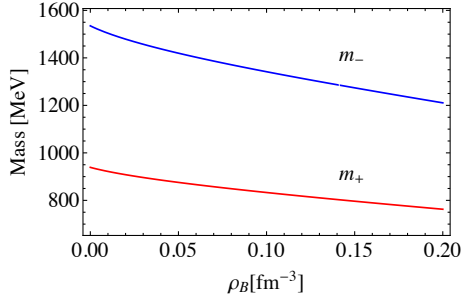
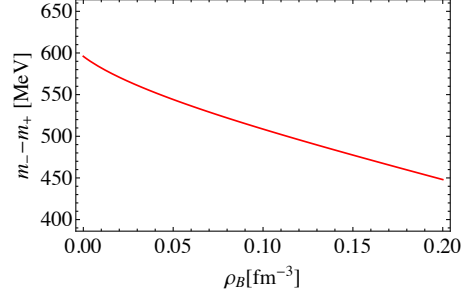
(a) Mass of the nucleon and $N^*(1535)$ (b) Mass difference of $N^*(1535)$ and the nucleon

FIG. 2. (color online) Density dependence of (a) masses of the nucleon (m_+) (red curve) and $N^*(1535)$ (m_-) (blue curve), and (b) mass difference between $N^*(1535)$ and the nucleon for $m_0 = 500$ [MeV]. Mass of the nucleon decreases gradually while that of $N^*(1535)$ decreases more rapidly, so that mass difference between $N^*(1535)$ and the nucleon gets small as the density increases.

k_{F+} and k_{F-} are the Fermi momentum for the nucleon and $N^*(1535)$ which are defined by $\mu_B = \sqrt{k_{F+}^2 + m_+^2}$ and $\mu_B = \sqrt{k_{F-}^2 + m_-^2}$, respectively. Note that in obtaining Eq. (29), we have subtracted the momentum integral which is not dependent on the Fermi momentum k_{F+} or k_{F-} explicitly [16]. The baryon number density ρ_B is defined by

$$\rho_B = \frac{2}{3\pi^2} k_{F+}^3 + \frac{2}{3\pi^2} k_{F-}^3. \quad (30)$$

The solution of σ_0 for $m_0 = 500$ [MeV] which satisfies Eq. (29) determines the density dependence of mean field σ_0 as shown in Fig. 1. As we can see, mean field σ_0 decreases as the density increases, which shows the tendency of partial restoration of chiral symmetry. Density dependence of masses of the nucleon and $N^*(1535)$, and mass difference between them, for $m_0 = 500$ [MeV] are plotted in Fig. 2. Red and blue curves in Fig. 2 (a) represent masses of the nucleon and $N^*(1535)$, respectively. This figure shows that mass of the nucleon decreases gradually while that of $N^*(1535)$ decreases more rapidly, and as a result, mass difference between $N^*(1535)$ and the nucleon gets small as the density increases as shown in Fig. 2 (b).

B. Fluctuations of pion and η meson

In order to study the decay property of $N^*(1535)$ in the nuclear matter, we need to get the propagators of fluctuation of pion ($\tilde{G}_\pi(q_0, \vec{q})$) and η meson ($\tilde{G}_\eta(q_0, \vec{q})$) on the background of mean field σ_0 as will be shown later. From the point of view of chiral symmetry, we need to consider the fluctuation of σ meson as well. In the present study, however, we assume that the decay width

of $N^*(1535)$ is dominated by $\Gamma_{N^* \rightarrow N\pi}$ and $\Gamma_{N^* \rightarrow N\eta}$ as in Eqs. (16) and (17). Hence, three body decay as $\Gamma_{N^* \rightarrow N\pi\pi}$ is neglected and the same is true for $\Gamma_{N^* \rightarrow N\sigma}$.

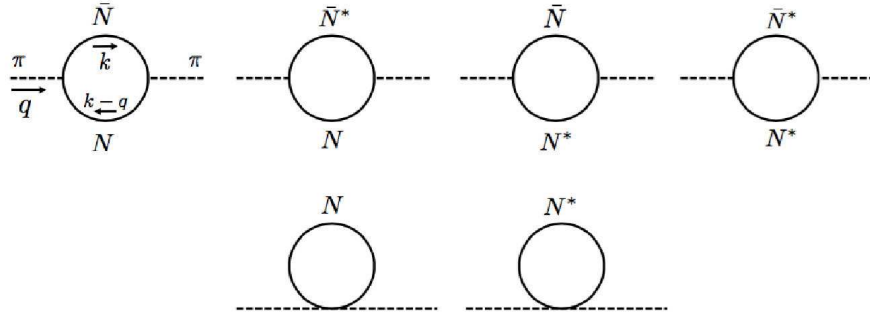
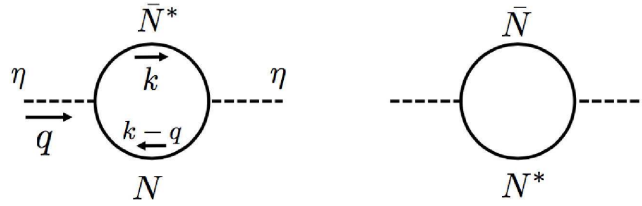
The propagators of pion and η meson are derived by the inverse of second functional derivative of the effective action $\Gamma[\sigma_0 + \sigma, \pi_r, \eta]$ in Eq. (26) with respect to π^a or η :

$$\begin{aligned} \tilde{G}_\pi^{ab}(q_0, \vec{q}) &= i \left(\int d^4x e^{iq \cdot x} \frac{\delta}{\delta \pi_r^a(x)} \frac{\delta}{\delta \pi_r^b(0)} \Gamma \right)^{-1} \\ &\equiv \frac{i \delta^{ab}}{q^2 - m_\pi^2 - i \tilde{\Sigma}_\pi(q_0, \vec{q})}, \end{aligned} \quad (31)$$

$$\begin{aligned} \tilde{G}_\eta(q_0, \vec{q}) &= i \left(\int d^4x e^{iq \cdot x} \frac{\delta}{\delta \eta(x)} \frac{\delta}{\delta \eta(0)} \Gamma \right)^{-1} \\ &\equiv \frac{i}{q^2 - m_\eta^2 - i \tilde{\Sigma}_\eta(q_0, \vec{q})}, \end{aligned} \quad (32)$$

respectively, where the “bare” mass m_π in Eq. (31) is given by Eq. (15), and m_η in Eq. (32) is fixed as $m_\eta = 547$ [MeV]. The self-energies $\tilde{\Sigma}_\pi(q_0, \vec{q})$ and $\tilde{\Sigma}_\eta(q_0, \vec{q})$ are diagrammatically shown in Fig. 3 and Fig. 4, respectively. We give the detailed expression of $\tilde{\Sigma}_\pi(q_0, \vec{q})$ and $\tilde{\Sigma}_\eta(q_0, \vec{q})$ in Appendix. A. Note that we should utilize propagators of pion which is of the form in Eq. (31) as fluctuation on the background of σ_0 in order to make our discussion consistent with the gap equation in Eq. (29), as pointed in Appendix. A.

It is possible to calculate the decay width of $N^*(1535)$ in nuclear matter by utilizing the propagators obtained in Eqs. (31) and (32). In the present analysis, however, we employ a more useful method which is called spectral representation method [33]. In this method, the decay width of $N^*(1535)$ is calculated by means of spectral functions for pion and η meson: $\rho_\pi(q_0, \vec{q})$ and $\rho_\eta(q_0, \vec{q})$, in nuclear matter, so that we need to obtain them. These are derived by the imaginary part of retarded Green’s functions

FIG. 3. Self-energy for pion ($\tilde{\Sigma}_\pi(q_0, \vec{q})$)FIG. 4. Self-energy for η meson ($\tilde{\Sigma}_\eta(q_0, \vec{q})$)

which include the infinite sums of self-energies as in the propagators in Eqs. (31) and (32):

$$\begin{aligned}
 & \rho_{\pi(\eta)}(q_0, \vec{q}) \\
 &= 2\text{Im}\tilde{G}_{\pi(\eta)}^R(q_0, \vec{q}) \\
 &= 2\text{Im}\left[-\frac{1}{q^2 - m_{\pi(\eta)}^2 - \tilde{\Sigma}_{\pi(\eta)}^R(q_0, \vec{q})}\right] \\
 &= \frac{-2\text{Im}\tilde{\Sigma}_{\pi(\eta)}^R(q_0, \vec{q})}{\left[q^2 - m_{\pi(\eta)}^2 - \text{Re}\tilde{\Sigma}_{\pi(\eta)}^R(q_0, \vec{q})\right]^2 + \left[\text{Im}\tilde{\Sigma}_{\pi(\eta)}^R(q_0, \vec{q})\right]^2}. \quad (33)
 \end{aligned}$$

Thanks to the charge conjugation invariance for pion and η meson, the retarded self-energy $\tilde{\Sigma}_{\pi(\eta)}^R(q_0, \vec{q})$ and the self-energy $\tilde{\Sigma}_{\pi(\eta)}(q_0, \vec{q})$ is related as

$$\text{Re}\tilde{\Sigma}_{\pi(\eta)}^R(q_0, \vec{q}) = \text{Re}\left(i\tilde{\Sigma}_{\pi(\eta)}(q_0, \vec{q})\right), \quad (34)$$

$$\text{Im}\tilde{\Sigma}_{\pi(\eta)}^R(q_0, \vec{q}) = \epsilon(q_0)\text{Im}\left(i\tilde{\Sigma}_{\pi(\eta)}(q_0, \vec{q})\right). \quad (35)$$

By utilizing the spectral functions for pion and η meson in Eq. (33), we can calculate the decay width of $N^*(1535)$ in nuclear matter in Sec. IV.

C. Excitation of η meson mode and particle-hole mode in nuclear matter

Here, we shall show appearance of a particle-hole mode (p - h mode) as a one particle state in addition to the η

meson mode in the propagator of η meson obtained by Eq. (32). The p - h mode plays an important role in calculating the modification of decay width of $N^*(1535)$ as will be explained in Sec. IV.

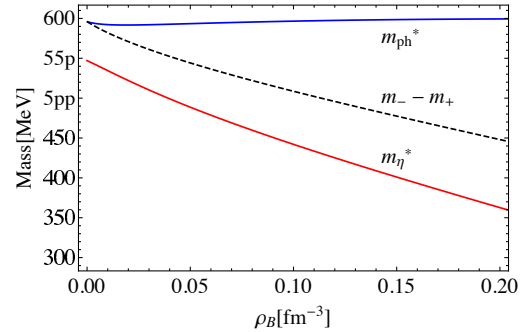


FIG. 5. (color online) Two solutions of Eq. (36) with $\vec{q} = \vec{0}$. Red curve is regarded as the mass of η meson mode since this curve reaches $q_0 = 547$ [MeV] at $\rho_B = 0$ [fm^{-3}]. Blue one is identified as the mass of p - h mode. Dashed curve is mass difference between $N^*(1535)$ and the nucleon, which is plotted as a reference.

One particle state of the propagator of η meson is defined by a solution q_0 of the following equation:

$$\text{Re}\left(i\tilde{G}_\eta^{-1}(q_0, \vec{q})\right) = q^2 - m_\eta^2 - \text{Re}\left(i\tilde{\Sigma}_\eta(q_0, \vec{q})\right) = 0. \quad (36)$$

This equation possesses two solutions which are identified as η meson mode and p - h mode, respectively. We plot

the density dependence of these solutions with $\vec{q} = \vec{0}$ which can be referred as mass of each modes, in Fig. 5. Red curve is regarded as the η meson mode since this curve reaches $q_0 = 547$ [MeV] at $\rho_B = 0$ [fm $^{-3}$]. Blue one is identified as the p - h mode. Dashed curve is mass difference between $N^*(1535)$ and the nucleon: $\Delta_m \equiv m_- - m_+$ which is plotted as a reference. We should note that the mass of η meson mode decreases as the density increases, and this curve always lies below Δ_m . On the other hand, mass of p - h mode is stable against the density, and this mode is always above Δ_m .

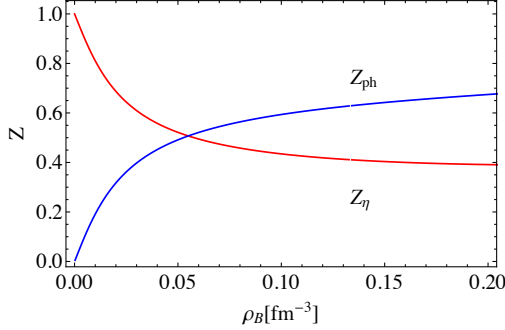


FIG. 6. (color online) Density dependence of Z_η and Z_{ph} defined by Eq. (37). Red curve is Z_η (η meson mode), and blue one is Z_{ph} (p - h mode).

In order to study the relative strength of η meson mode and p - h mode, it is worth calculating Z -factors of them. They are defined by

$$Z_i^{-1} \equiv \frac{1}{2m_\eta} \frac{\partial}{\partial q_0} \left(q_0^2 - m_\eta^2 - \text{Re}(i\tilde{\Sigma}_\eta(q_0, \vec{0})) \right)_{q_0=m_i^*} . \quad (37)$$

The subscript i runs over $i = \eta, ph$, and m_η^* stands for the mass of η meson mode while m_{ph}^* stands for the mass of p - h mode. $\frac{1}{2m_\eta}$ in front of the derivative in Eq. (37) is added as a normalization. We plot density dependence of Z_η and Z_{ph} in Fig. 6. This figure shows that Z_η starts from $Z_\eta = 1$ at $\rho_B = 0$ [fm $^{-3}$], and decreases as the density increases. On the other hand, Z_{ph} starts from $Z_{ph} = 0$ at $\rho_B = 0$ [fm $^{-3}$], and increases. As a result, Z_{ph} gets larger than Z_η around $\rho_B \sim 0.055$ [fm $^{-3}$], which means the strength of p - h mode is stronger than that of η meson mode at higher density. This inversion will play a significant role in calculating the decay width of $N^*(1535)$ in Sec. IV.

IV. CALCULATIONS AND RESULTS

In this section, we calculate the modification of decay width of $N^*(1535)$ in the nuclear matter constructed in Sec. III. In Sec. IV A, we show the way to calculate the decay width by means of the spectral functions obtained

in Eq. (33) in detail, by computing $N\eta$ loop as an example. In Sec. IV B, we indicate density dependence of total decay width of $N^*(1535)$: Γ_{tot} , and partial decay widths of $\Gamma_{N^* \rightarrow N\pi}$ and $\Gamma_{N^* \rightarrow N\eta}$.

A. Calculation method

The decay width of $N^*(1535)$ is given by the imaginary part of self-energies for $N^*(1535)$ shown in Fig. 7 from the Cutkosky rule. As mentioned in Sec. III B, one of the most useful way to calculate it is the spectral representation method [33]. Here, we shall show how to utilize this method to calculate the imaginary part of self-energy for $N^*(1535)$ in Fig. 7 (a) as an example. The blob in Fig. 7 indicates the infinite sums of self-energies for η meson or pion as explained in Sec. III B.

When we define a greater self-energy $\Sigma_{N^*(a)}^>(x_0, \vec{x})$ and a lesser self-energy $\Sigma_{N^*(a)}^<(x_0, \vec{x})$ through the self-energy $\Sigma_{N^*(a)}(x_0, \vec{x})$ in Fig. 7 (a) as

$$\Sigma_{N^*(a)}(x_0, \vec{x}) = \theta(x_0) \Sigma_{N^*(a)}^>(x_0, \vec{x}) + \theta(-x_0) \Sigma_{N^*(a)}^<(x_0, \vec{x}) , \quad (38)$$

and define a retarded self-energy $\Sigma_{N^*(a)}^R(x_0, \vec{x})$ by

$$\Sigma_{N^*(a)}^R(x_0, \vec{x}) = i\theta(x_0) \left(\Sigma_{N^*(a)}^>(x_0, \vec{x}) - \Sigma_{N^*(a)}^<(x_0, \vec{x}) \right) , \quad (39)$$

it is well known that the following relation holds [33]:

$$\overline{\text{Im}} \tilde{\Sigma}_{N^*(a)}^R(q_0, \vec{q}) = \frac{1}{2} \left(\tilde{\Sigma}_{N^*(a)}^>(q_0, \vec{q}) - \tilde{\Sigma}_{N^*(a)}^<(q_0, \vec{q}) \right) . \quad (40)$$

In Eq. (40), $\tilde{\Sigma}_{N^*(a)}^R(q_0, \vec{q})$, $\tilde{\Sigma}_{N^*(a)}^>(q_0, \vec{q})$ and $\tilde{\Sigma}_{N^*(a)}^<(q_0, \vec{q})$ are the Fourier transformation of $\Sigma_{N^*(a)}^R(x_0, \vec{x})$, $\Sigma_{N^*(a)}^>(x_0, \vec{x})$ and $\Sigma_{N^*(a)}^<(x_0, \vec{x})$, respectively, and the symbol $\overline{\text{Im}}X$ is defined by

$$\overline{\text{Im}}X \equiv \frac{X - \gamma_0 X^\dagger \gamma_0}{2i} . \quad (41)$$

According to Eq. (40), we need to find explicit forms of $\tilde{\Sigma}_{N^*(a)}^>(q_0, \vec{q})$ and $\tilde{\Sigma}_{N^*(a)}^<(q_0, \vec{q})$ to evaluate $\overline{\text{Im}} \tilde{\Sigma}_{N^*(a)}^R(q_0, \vec{q})$, so that we shall give them next. When we define a greater Green's function $G_\eta^>(x_0, \vec{x})$ ($S_\eta^>(x_0, \vec{x})$) and a lesser Green's function $G_\eta^<(x_0, \vec{x})$ ($S_\eta^<(x_0, \vec{x})$) for η meson (the nucleon) by

$$\begin{aligned} G_\eta(x_0, \vec{x}) &= \theta(x_0) G_\eta^>(x_0, \vec{x}) + \theta(-x_0) G_\eta^<(x_0, \vec{x}) , \\ S_N(x_0, \vec{x}) &= \theta(x_0) S_N^>(x_0, \vec{x}) + \theta(-x_0) S_N^<(x_0, \vec{x}) , \end{aligned} \quad (42)$$

the self-energy for $N^*(1535)$ in Fig. 7 (a) is expressed as

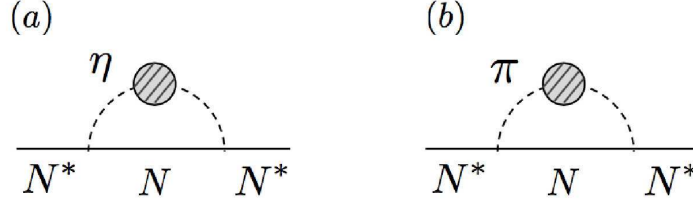


FIG. 7. (color online) Self-energies for $N^*(1535)$ from (a) $N\eta$ loop and (b) $N\pi$ loop. The blobs indicate the infinite sums of self-energies for η meson or pion as explained in Sec. III B.

$$\begin{aligned}\Sigma_{N^*(a)}(x_0, \vec{x}) &= (ig_{NN^*\eta})S_N(x_0, \vec{x})(ig_{NN^*\eta}^*)G_\eta(x_0, \vec{x}) \\ &= (ig_{NN^*\eta})\left(\theta(x_0)S_N^>(x_0, \vec{x}) + \theta(-x_0)S_N^<(x_0, \vec{x})\right)(ig_{NN^*\eta}^*)\left(\theta(x_0)G_\eta^>(x_0, \vec{x}) + \theta(-x_0)G_\eta^<(x_0, \vec{x})\right) \\ &= \theta(x_0)(ig_{NN^*\eta})S_N^>(x_0, \vec{x})(ig_{NN^*\eta}^*)G_\eta^>(x_0, \vec{x}) + \theta(-x_0)(ig_{NN^*\eta})S_N^<(x_0, \vec{x})(ig_{NN^*\eta}^*)G_\eta^<(x_0, \vec{x}).\end{aligned}\quad (43)$$

Eq. (43) together with Eq. (38) reads

$$\Sigma_{N^*(a)}^>(x_0, \vec{x}) = (ig_{NN^*\eta})S_N^>(x_0, \vec{x})(ig_{NN^*\eta}^*)G_\eta^>(x_0, \vec{x}), \quad (44)$$

$$\Sigma_{N^*(a)}^<(x_0, \vec{x}) = (ig_{NN^*\eta})S_N^<(x_0, \vec{x})(ig_{NN^*\eta}^*)G_\eta^<(x_0, \vec{x}), \quad (45)$$

so that $\text{Im}\tilde{\Sigma}_{N^*(a)}^R(q_0, \vec{q})$ in Eq. (40) is calculated as

$$\begin{aligned}\text{Im}\tilde{\Sigma}_{N^*(a)}^R(q_0, \vec{q}) &= \frac{1}{2} \int \frac{d^4k}{(2\pi)^4} \left(F(\vec{k}; \Lambda)\right)^2 \left\{ (ig_{NN^*\eta})\tilde{S}_N^>(k_0, \vec{k})(ig_{NN^*\eta}^*)\tilde{G}_\eta^>(q_0 - k_0, \vec{q} - \vec{k}) \right. \\ &\quad \left. - (ig_{NN^*\eta})\tilde{S}_N^<(k_0, \vec{k})(ig_{NN^*\eta}^*)\tilde{G}_\eta^<(q_0 - k_0, \vec{q} - \vec{k}) \right\}.\end{aligned}\quad (46)$$

In obtaining Eq. (46), we have inserted the form factor $F(\vec{k}; \Lambda)$ defined by Eq. (20) to take into account the hadron size. The value of cutoff parameter Λ is chosen to be $\Lambda = 300$ [MeV] in the present analysis, which is slightly higher than that of Fermi momentum at normal nuclear matter density [16].

Furthermore, the Fourier transformation of the greater Green's function $\tilde{G}_\eta^>(q_0, \vec{q})$ and the lesser Green's function $\tilde{G}_\eta^<(q_0, \vec{q})$ for η meson are related to the spectral function for η meson $\rho_\eta(q_0, \vec{q})$ in an equilibrium system by following relations [33]:

$$\tilde{G}_\eta^>(q_0, \vec{q}) = (1 + f(q_0))\rho_\eta(q_0, \vec{q}) \quad (47)$$

$$\tilde{G}_\eta^<(q_0, \vec{q}) = f(q_0)\rho_\eta(q_0, \vec{q}), \quad (48)$$

where $f(q_0)$ is the Bose-Einstein distribution function. In a similar way, $\tilde{S}_N^>(q_0, \vec{q})$ and $\tilde{S}_N^<(q_0, \vec{q})$ are related to spectral function for the nucleon $\rho_N(q_0, \vec{q})$ as

$$\tilde{S}_N^>(q_0, \vec{q}) = (1 - \tilde{f}(q_0 - \mu_B))\rho_N(q_0, \vec{q}) \quad (49)$$

$$\tilde{S}_N^<(q_0, \vec{q}) = -\tilde{f}(q_0 - \mu_B)\rho_N(q_0, \vec{q}), \quad (50)$$

where $\tilde{f}(q_0 - \mu_B)$ is the Fermi-Dirac distribution function.

Note that minus signs in Eqs. (49) and (50) reflect the Pauli blocking of Fermions, and we have not given the baryon chemical potential to distribution function for η meson in Eqs. (47) and (48) since η meson does not have a baryon number. In the present study, $\rho_\eta(q_0, \vec{q})$ is obtained by Eq. (33) and $\rho_N(q_0, \vec{q})$ is of the form

$$\rho_N(q_0, \vec{q}) = 2\pi(\not{q} + m_+)\epsilon(q_0)\delta(q^2 - m_+^2). \quad (51)$$

Bose-Einstein distribution function $f(q_0)$ and Fermi-Dirac distribution function $\tilde{f}(q_0 - \mu_B)$ at zero temperature takes the form

$$f(q_0) = \frac{1}{e^{q_0/T} - 1} \xrightarrow{T \rightarrow 0} -\theta(-q_0), \quad (52)$$

$$\tilde{f}(q_0 - \mu_B) = \frac{1}{e^{(q_0 - \mu_B)/T} + 1} \xrightarrow{T \rightarrow 0} \theta(\mu_B - q_0). \quad (53)$$

Utilizing Eqs. (47) - (53), the imaginary part of self-energy for $N^*(1535)$ in Fig. 7 (a) in Eq. (46) is finally evaluated as

$$\text{Im}\tilde{\Sigma}_{N^*(a)}^R(q_0, \vec{q}) = \frac{1}{2} \int \frac{d^4k}{(2\pi)^3} \left(F(\vec{k}; \Lambda)\right)^2 (\theta(q_0 - k_0) - \theta(\mu_B - k_0))$$

$$\times (ig_{NN^*\eta})(\not{k} + m_+)\epsilon(k_0)\delta(k^2 - m_+^2)(ig_{NN^*\eta}^*)\rho_\eta(q_0 - k_0, \vec{q} - \vec{k}). \quad (54)$$

Then, the partial decay width of $\Gamma_{N^* \rightarrow N\eta}$ in nuclear matter is computed by summing up the spin and isospin for final state, and taking the spin and isospin average for initial state of $N^*(1535)$ as

$$\Gamma_{N^* \rightarrow N\eta} = \frac{1}{2m_-} \text{Tr} \left[(\not{q} + m_+) \overline{\text{Im}} \tilde{\Sigma}_{N^*(a)}^R(q_0, \vec{q}) \right]. \quad (55)$$

In Eq. (55), the momentum for initial state of $N^*(1535)$ satisfies the on-shell condition: $q^2 = m_+^2$. In a similar way, partial decay width of $\Gamma_{N^* \rightarrow N\pi}$ and total width are also obtained.

B. Results

The resultant density dependence of the total decay width of $N^*(1535)$ (Γ_{tot}) in nuclear matter for $m_0 = 500$ [MeV], $m_0 = 700$ [MeV] and $m_0 = 900$ [MeV] are plotted in Fig. 8. Red circles are the results, and dashed red line is the total width in the vacuum $\Gamma_{\text{tot}}^{\text{vac}} = 150$ [MeV] which is added as a reference. For any choice of the value of m_0 , total decay width of $N^*(1535)$ gets small at density, and drops to about 100 [MeV] at normal nuclear matter density $\rho_0 = 0.16$ [fm $^{-3}$]. This tendency shows that decay width of $N^*(1535)$ gets closed in nuclear matter although a naive expectation leads to the broadening of decay width due to collisions with nucleons surrounding $N^*(1535)$.

The origin of this narrowing can be understood well if we investigate the density dependence of partial width of $\Gamma_{N^* \rightarrow N\pi}$ and $\Gamma_{N^* \rightarrow N\eta}$. These partial widths for $m_0 = 500$ [MeV] are indicated in Fig. 9. Red circles represent the partial width of $\Gamma_{N^* \rightarrow N\pi}$ and blue circles are $\Gamma_{N^* \rightarrow N\eta}$. Purple dashed line is the vacuum values of them: $\Gamma_{N^* \rightarrow N\pi}^{\text{vac}} = \Gamma_{N^* \rightarrow N\eta}^{\text{vac}} = 75$ [MeV], which is shown as a reference. As we can see, the partial width of $\Gamma_{N^* \rightarrow N\pi}$ is broadened as we expect while that of $\Gamma_{N^* \rightarrow N\eta}$ is drastically closed as the density increases. This unexpected behavior of $\Gamma_{N^* \rightarrow N\eta}$ is explained as following. Although a decay process of $N^* \rightarrow N + (\eta \text{ meson mode})$ in nuclear matter is allowed since phase space is not closed as we can see in Fig. 5 (m_η^* is always below $m_- - m_+$), the Z -factor for η meson mode (Z_η) is converted into that of p - h mode (Z_{ph}) as shown in Fig. 6. The mass of p - h mode is always above the mass difference $m_- - m_+$, so that imaginary part from decay process of $N^* \rightarrow N + (p$ - $h \text{ mode})$ is not generated. Therefore, main part of imaginary part in Fig. 7 (b) is lost and resulting partial width is suppressed as shown in Fig. 9.

These modifications of decay properties, especially the drastic narrowing of the partial width of $\Gamma_{N^* \rightarrow N\eta}$, together with the dropping of mass of $N^*(1535)$ provide experiments for observing the chiral restoration in nuclear matter by means of $N^*(1535)$ resonance with useful

information.

V. CONCLUSIONS

In the present study, we investigate the mass and decay width of $N^*(1535)$ in nuclear matter to give some clues to understand the partial restoration of chiral symmetry in medium. The nucleon and $N^*(1535)$ are introduced within the parity doublet model, so that the nucleon and $N^*(1535)$ is regarded as the chiral partner to each other. Then, mass difference between $N^*(1535)$ and the nucleon is expected to get small as the density increases in which chiral restoration is realized.

In this study, we determine model parameters from Ref. [29] in which properties of nuclear matter at normal nuclear matter density is reproduced by means of a mean field approach. While only non-derivative coupling are included in this reference, we add derivative couplings as well in order to explain a large value of width of $N^*(1535)$ in the vacuum. This procedure does not change properties of nuclear matter since spatially homogeneous mean field is considered in Ref. [29], so that derivative of the mean field vanishes. We also include $NN^*\eta$ coupling as done in Ref. [18].

In Fig. 1, we plot density dependence of mean field of σ meson with chiral invariant mass m_0 being 500 [MeV]. As the density increases, the value of mean field decreases which shows a tendency of partial restoration of chiral symmetry in nuclear matter. Accordingly, mass of the nucleon decreases gradually while that of $N^*(1535)$ decreases more rapidly so that mass difference between $N^*(1535)$ and the nucleon drops as the density increases, as we can see in Fig. 2.

The decay width of $N^*(1535)$ in nuclear matter is studied by calculating the imaginary part of self-energy for $N^*(1535)$ in Fig. 7. In this figure, propagators of pion and η meson should be ones which include infinite sums of self-energies in Fig. 3 and Fig. 4 to be consistent with the gap equation in Eq. (29). The calculations show the width of $N^*(1535)$ is suppressed at density as shown in Fig. 8, in contrast to the naive expectation in which collisional broadening provides an enlargement of width of $N^*(1535)$. This behavior is caused by the drastic suppression of partial decay width of $\Gamma_{N^* \rightarrow N\eta}$ as we can see in Fig. 9. Although the decay process of $N^* \rightarrow N + (\eta \text{ meson mode})$ is allowed since phase space is not closed, the Z -factor for η meson is converted into that of p - h mode as we access to finite density as shown in Fig. 5 and Fig. 6. The decay process of $N^* \rightarrow N + (p$ - $h \text{ mode})$ is always zero since its phase space is not opened at any densities. Therefore, main part of imaginary part in Fig. 7 (b) is lost and resulting partial width is drastically suppressed. These modifications of decay properties of

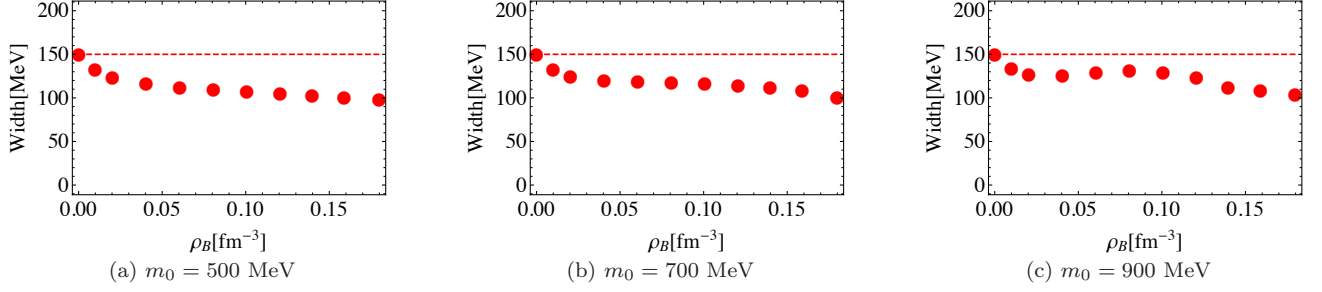


FIG. 8. (color online) Density dependence of total width of $N^*(1535)$: Γ_{tot} , for (a) $m_0 = 500$ [MeV], (b) $m_0 = 700$ [MeV], (c) $m_0 = 900$ [MeV]. Red circles are the results, and dashed red line is the total width in the vacuum $\Gamma_{\text{tot}}^{\text{vac}} = 150$ [MeV] which is added as a reference.

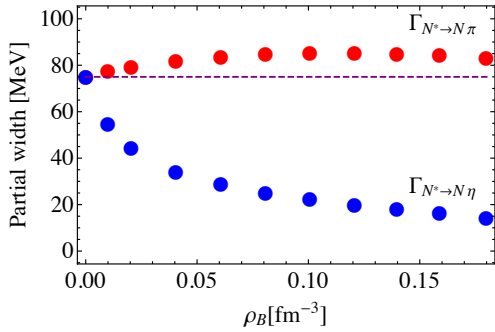


FIG. 9. (color online) Density dependence of partial width of $N^*(1535)$ for $m_0 = 500$ [MeV]. Red circles represent the partial width of $\Gamma_{N^* \rightarrow N\pi}$ and blue circles are $\Gamma_{N^* \rightarrow N\eta}$. Purple dashed line is the vacuum values of them: $\Gamma_{N^* \rightarrow N\pi}^{\text{vac}} = \Gamma_{N^* \rightarrow N\eta}^{\text{vac}} = 75$ [MeV], which is shown as a reference.

$N^*(1535)$, especially the drastic narrowing of the partial width of $\Gamma_{N^* \rightarrow N\eta}$, together with the dropping of mass of $N^*(1535)$ provide experiments for observing the chiral restoration in nuclear matter by means of $N^*(1535)$ resonance with useful information.

In the following, we give some discussions which cannot be covered in this paper. In the present study, we calculate the decay width of $N^*(1535)$ in nuclear matter. In the experiment, $N^*(1535)$ is produced as a resonance state so that information of off-shell state is significant as well. From this point of view, it is interesting to study the spectral function of $N^*(1535)$ in nuclear matter, and we leave this in future publication.

In this study, we include η meson by an ad hoc way

into the parity doublet model since chiral symmetry is explicitly broken for strange sector due to the large mass of strange quark, and we assume that the “bare” mass of η meson is not changed from $m_\eta = 547$ [MeV] even at finite density region. It is possible to extend parity doublet model for three flavors and take into account a relation between the mass change of η meson and chiral symmetry. We expect, however, this effect for the decay width of $N^*(1535)$ does not provide significant change since Z -factor for η meson mode is suppressed in nuclear matter as we have found in the present analysis.

Furthermore, we assume that three body decay of $N^*(1535)$ vanishes and we ignore sigma meson decay processes. We have confirmed that such sigma meson decay provides the decay width of $N^*(1535)$ with corrections of only a few MeV at most.

ACKNOWLEDGMENTS

D.S. would like to thank to Y. Takeda for useful discussions on parity doublet model.

Appendix A: Self-energies for σ meson and pion

Here, we give explicit forms of self-energies for pion and η meson in Fig. 3 and Fig. 4. In calculating these diagrams, we employ the in-medium propagators for the nucleon and $N^*(1535)$:

$$\tilde{G}_{N_+}(k_0, \vec{k}) = (\not{k} + m_+) \left[\frac{i}{k^2 - m_+^2 + i\epsilon} - 2\pi\theta(k_0)\theta(k_{F+} - |\vec{k}|)\delta(k^2 - m_+^2) \right], \quad (\text{A1})$$

$$\tilde{G}_{N_-}(k_0, \vec{k}) = (\not{k} + m_-) \left[\frac{i}{k^2 - m_-^2 + i\epsilon} - 2\pi\theta(k_0)\theta(k_{F-} - |\vec{k}|)\delta(k^2 - m_-^2) \right]. \quad (\text{A2})$$

The resulting retarded self-energies for pion in Fig. (3) are obtained as

$$\begin{aligned}
\text{Re}\tilde{\Sigma}_\pi^R(q_0, \vec{q}) = & \frac{g_{NN\pi}^2}{2\pi^2} \int_0^{k_{F+}} d|\vec{k}| \frac{|\vec{k}|^2}{E_k^+} \left\{ 4 - \frac{q_0^2 - |\vec{q}|^2}{2|\vec{k}||\vec{q}|} \ln |A_+| \right\} - m_+^2 \frac{(2\sigma_0 h_{NN\pi})^2}{\pi^2} \int_0^{k_{F+}} d|\vec{k}| \frac{|\vec{k}|^2}{E_k^+} \frac{q_0^2 - |\vec{q}|^2}{|\vec{k}||\vec{q}|} \ln |A_+| \\
& + \frac{4\sigma_0 m_+ g_{NN\pi} h_{NN\pi}}{\pi^2} (q_0^2 - |\vec{q}|^2) \int_0^{k_{F+}} d|\vec{k}| \frac{|\vec{k}|^2}{E_k^+} \frac{1}{2|\vec{k}||\vec{q}|} \ln |A_+| \\
& + \frac{g_{NN^*\pi}^2}{\pi^2} \int_0^{k_{F+}} d|\vec{k}| \frac{|\vec{k}|^2}{E_k^+} \left\{ 2 - \frac{q_0^2 - |\vec{q}|^2 - (m_+ + m_-)^2}{4|\vec{k}||\vec{q}|} \ln |A_{+-}| \right\} \\
& + \frac{g_{NN^*\pi}^2}{\pi^2} \int_0^{k_{F-}} d|\vec{k}| \frac{|\vec{k}|^2}{E_k^-} \left\{ 2 - \frac{q_0^2 - |\vec{q}|^2 - (m_+ + m_-)^2}{4|\vec{k}||\vec{q}|} \ln |A_{-+}| \right\} \\
& + \frac{2(2\sigma_0 h_{NN^*\pi})^2}{\pi^2} \int_0^{k_{F+}} d|\vec{k}| \frac{|\vec{k}|^2}{2E_k^+} \left\{ 2(m_-^2 - m_+^2) - \frac{(m_- - m_+)(q_0^2 - |\vec{q}|^2 - (m_+ + m_-)^2)}{4|\vec{k}||\vec{q}|} \ln |A_{+-}| \right\} \\
& + \frac{2(2\sigma_0 h_{NN^*\pi})^2}{\pi^2} \int_0^{k_{F-}} d|\vec{k}| \frac{|\vec{k}|^2}{2E_k^-} \left\{ 2(m_-^2 - m_+^2) - \frac{(m_- - m_+)(q_0^2 - |\vec{q}|^2 - (m_+ + m_-)^2)}{4|\vec{k}||\vec{q}|} \ln |A_{-+}| \right\} \\
& + \frac{8\sigma_0 g_{NN^*\pi} h_{NN^*\pi}}{\pi^2} \int_0^{k_{F+}} d|\vec{k}| \frac{|\vec{k}|^2}{2E_k^+} \left\{ 2(m_+ + m_-) - \frac{(m_- - m_+)(q_0^2 - |\vec{q}|^2 - (m_+ + m_-)^2)}{4|\vec{k}||\vec{q}|} \right\} \ln |A_{+-}| \\
& - \frac{8\sigma_0 g_{NN^*\pi} h_{NN^*\pi}}{\pi^2} \int_0^{k_{F-}} d|\vec{k}| \frac{|\vec{k}|^2}{2E_k^-} \left\{ 2(m_+ + m_-) + \frac{(m_- - m_+)(q_0^2 - |\vec{q}|^2 - (m_+ + m_-)^2)}{4|\vec{k}||\vec{q}|} \right\} \ln |A_{-+}| \\
& + \frac{g_{N^*N^*\pi}^2}{2\pi^2} \int_0^{k_{F-}} d|\vec{k}| \frac{|\vec{k}|^2}{E_k^-} \left\{ 4 - \frac{q_0^2 - |\vec{q}|^2}{2|\vec{k}||\vec{q}|} \ln |A_-| \right\} - m_-^2 \frac{(2\sigma_0 h_{N^*N^*\pi})^2}{\pi^2} \int_0^{k_{F-}} d|\vec{k}| \frac{|\vec{k}|^2}{E_k^-} \frac{q_0^2 - |\vec{q}|^2}{|\vec{k}||\vec{q}|} \ln |A_-| \\
& + \frac{4\sigma_0 m_- g_{N^*N^*\pi} h_{N^*N^*\pi}}{\pi^2} (q_0^2 - |\vec{q}|^2) \int_0^{k_{F-}} d|\vec{k}| \frac{|\vec{k}|^2}{E_k^-} \frac{1}{2|\vec{k}||\vec{q}|} \ln |A_-| \\
& - \frac{2g_{NN\sigma} m_+}{\pi^2 \sigma_0} \int_0^{k_{F+}} d|\vec{k}| \frac{|\vec{k}|^2}{E_k^+} - \frac{2g_{N^*N^*\sigma} m_-}{\pi^2 \sigma_0} \int_0^{k_{F-}} d|\vec{k}| \frac{|\vec{k}|^2}{E_k^-}, \tag{A3}
\end{aligned}$$

and

$$\begin{aligned}
\text{Im}\tilde{\Sigma}_\pi^R(q_0, \vec{q}) = & \pi G_1^2 \frac{q_0^2 - |\vec{q}|^2}{2} \int \frac{d^3k}{(2\pi)^3} \frac{1}{E_k^+ E_{k-q}^+} \theta(k_{F+} - |\vec{k}|) \delta(q_0 - E_k^+ - E_{k-q}^+) \\
& - \pi G_1^2 \frac{q_0^2 - |\vec{q}|^2}{2} \int \frac{d^3k}{(2\pi)^3} \frac{1}{E_k^+ E_{k-q}^+} \theta(k_{F+} - |\vec{k}|) \delta(q_0 + E_k^+ + E_{k-q}^+) \\
& - \pi G_1^2 \frac{q_0^2 - |\vec{q}|^2}{2} \int \frac{d^3k}{(2\pi)^3} \frac{1}{E_k^+ E_{k-q}^+} \theta(k_{F+} - |\vec{k}|) \delta(q_0 - E_k^+ + E_{k-q}^+) \\
& + \pi G_1^2 \frac{q_0^2 - |\vec{q}|^2}{2} \int \frac{d^3k}{(2\pi)^3} \frac{1}{E_k^+ E_{k-q}^+} \theta(k_{F+} - |\vec{k}|) \delta(q_0 + E_k^+ - E_{k-q}^+) \\
& + \pi G_2^2 \frac{q_0^2 - |\vec{q}|^2 - (m_+ + m_-)^2}{2} \int \frac{d^3k}{(2\pi)^3} \frac{1}{E_k^+ E_{k-q}^-} \theta(k_{F+} - |\vec{k}|) \delta(q_0 - E_k^+ - E_{k-q}^-) \\
& + \pi G_2^2 g^2 \frac{q_0^2 - |\vec{q}|^2 - (m_+ + m_-)^2}{2} \int \frac{d^3k}{(2\pi)^3} \frac{1}{E_k^- E_{k-q}^+} \theta(k_{F-} - |\vec{k}|) \delta(q_0 - E_k^- - E_{k-q}^+) \\
& - \pi G_2^2 \frac{q_0^2 - |\vec{q}|^2 - (m_+ + m_-)^2}{2} \int \frac{d^3k}{(2\pi)^3} \frac{1}{E_{k-q}^+ E_k^-} \theta(k_{F-} - |\vec{k}|) \delta(q_0 + E_{k-q}^+ + E_k^-) \\
& - \pi G_2^2 \frac{q_0^2 - |\vec{q}|^2 - (m_+ + m_-)^2}{2} \int \frac{d^3k}{(2\pi)^3} \frac{1}{E_{k-q}^- E_k^+} \theta(k_{F+} - |\vec{k}|) \delta(q_0 + E_{k-q}^- + E_k^+)
\end{aligned}$$

$$\begin{aligned}
& -\pi G_2^2 \frac{q_0^2 - |\vec{q}|^2 - (m_+ + m_-)^2}{2} \int \frac{d^3k}{(2\pi)^3} \frac{1}{E_k^+ E_{k-q}^-} \theta(k_{F+} - |\vec{k}|) \delta(q_0 - E_k^+ + E_{k-q}^-) \\
& -\pi G_2^2 \frac{q_0^2 - |\vec{q}|^2 - (m_+ + m_-)^2}{2} \int \frac{d^3k}{(2\pi)^3} \frac{1}{E_k^- E_{k-q}^+} \theta(k_{F-} - |\vec{k}|) \delta(q_0 - E_k^- + E_{k-q}^+) \\
& +\pi G_2^2 \frac{q_0^2 - |\vec{q}|^2 - (m_+ + m_-)^2}{2} \int \frac{d^3k}{(2\pi)^3} \frac{1}{E_{k-q}^+ E_k^-} \theta(k_{F-} - |\vec{k}|) \delta(q_0 - E_{k-q}^+ + E_k^-) \\
& +\pi G_2^2 \frac{q_0^2 - |\vec{q}|^2 - (m_+ + m_-)^2}{2} \int \frac{d^3k}{(2\pi)^3} \frac{1}{E_{k-q}^- E_k^+} \theta(k_{F+} - |\vec{k}|) \delta(q_0 - E_{k-q}^- + E_k^+) \\
& +\pi G_3^2 \frac{q_0^2 - |\vec{q}|^2}{2} \int \frac{d^3k}{(2\pi)^3} \frac{1}{E_k^- E_{k-q}^-} \theta(k_{F-} - |\vec{k}|) \delta(q_0 - E_k^- - E_{k-q}^-) \\
& -\pi G_3^2 \frac{q_0^2 - |\vec{q}|^2}{2} \int \frac{d^3k}{(2\pi)^3} \frac{1}{E_k^- E_{k-q}^-} \theta(k_{F-} - |\vec{k}|) \delta(q_0 + E_k^- + E_{k-q}^-) \\
& -\pi G_3^2 \frac{q_0^2 - |\vec{q}|^2}{2} \int \frac{d^3k}{(2\pi)^3} \frac{1}{E_k^- E_{k-q}^-} \theta(k_{F-} - |\vec{k}|) \delta(q_0 - E_k^- + E_{k-q}^-) \\
& +\pi G_3^2 \frac{q_0^2 - |\vec{q}|^2}{2} \int \frac{d^3k}{(2\pi)^3} \frac{1}{E_k^- E_{k-q}^-} \theta(k_{F-} - |\vec{k}|) \delta(q_0 + E_k^- - E_{k-q}^-) ,
\end{aligned} \tag{A4}$$

where we have defined

$$\begin{aligned}
E_k^+ & \equiv \sqrt{|\vec{k}|^2 + m_+^2} \\
E_k^- & \equiv \sqrt{|\vec{k}|^2 + m_-^2} ,
\end{aligned} \tag{A5}$$

and

$$A_+ \equiv \frac{q_0^2 - |\vec{q}|^2 + 2|\vec{k}||\vec{q}| + 2q_0 E_k^+}{q_0^2 - |\vec{q}|^2 - 2|\vec{k}||\vec{q}| + 2q_0 E_k^+} \frac{q_0^2 - |\vec{q}|^2 + 2|\vec{k}||\vec{q}| - 2q_0 E_k^+}{q_0^2 - |\vec{q}|^2 - 2|\vec{k}||\vec{q}| - 2q_0 E_k^+} , \tag{A6}$$

$$A_{+-} \equiv \frac{q_0^2 - |\vec{q}|^2 + 2|\vec{k}||\vec{q}| + m_+^2 - m_-^2 + 2q_0 E_k^+}{q_0^2 - |\vec{q}|^2 - 2|\vec{k}||\vec{q}| + m_+^2 - m_-^2 + 2q_0 E_k^+} \frac{q_0^2 - |\vec{q}|^2 + 2|\vec{k}||\vec{q}| + m_+^2 - m_-^2 - 2q_0 E_k^+}{q_0^2 - |\vec{q}|^2 - 2|\vec{k}||\vec{q}| + m_+^2 - m_-^2 - 2q_0 E_k^+} , \tag{A7}$$

$$A_{-+} \equiv \frac{q_0^2 - |\vec{q}|^2 - 2q_0 E_k^- + 2|\vec{k}||\vec{q}| + m_-^2 - m_+^2}{q_0^2 - |\vec{q}|^2 - 2q_0 E_k^- - 2|\vec{k}||\vec{q}| + m_-^2 - m_+^2} \frac{q_0^2 - |\vec{q}|^2 + 2q_0 E_k^- + 2|\vec{k}||\vec{q}| + m_-^2 - m_+^2}{q_0^2 - |\vec{q}|^2 + 2q_0 E_k^- - 2|\vec{k}||\vec{q}| + m_-^2 - m_+^2} , \tag{A8}$$

$$A_- \equiv \frac{q_0^2 - |\vec{q}|^2 + 2|\vec{k}||\vec{q}| + 2q_0 E_k^-}{q_0^2 - |\vec{q}|^2 - 2|\vec{k}||\vec{q}| + 2q_0 E_k^-} \frac{q_0^2 - |\vec{q}|^2 + 2|\vec{k}||\vec{q}| - 2q_0 E_k^-}{q_0^2 - |\vec{q}|^2 - 2|\vec{k}||\vec{q}| - 2q_0 E_k^-} . \tag{A9}$$

The couplings G_1 , G_2 and G_3 are defined by

$$G_1 \equiv g_{NN\pi} + 4\sigma_0 m_+ h_{NN\pi} , \tag{A10}$$

$$G_2 \equiv g_{NN^*\pi} + 2\sigma_0 (m_- - m_+) h_{NN^*\pi} , \tag{A11}$$

$$G_3 \equiv g_{N^*N^*\pi} + 4\sigma_0 m_- h_{N^*N^*\pi} . \tag{A12}$$

In a similar way, the self-energies for η meson in Fig. 4 are

$$\begin{aligned}
\text{Re}\tilde{\Sigma}_\eta^R(q_0, \vec{q}) &= \frac{g_{NN^*\eta}^2}{\pi^2} \int_0^{k_{F+}} d|\vec{k}| \frac{|\vec{k}|^2}{E_k^+} \left\{ 2 - \frac{q_0^2 - |\vec{q}|^2 - (m_+ + m_-)^2}{4|\vec{k}||\vec{q}|} \ln |A_{+-}| \right\} \\
&+ \frac{g_{NN^*\eta}^2}{\pi^2} \int_0^{k_{F-}} d|\vec{k}| \frac{|\vec{k}|^2}{E_k^-} \left\{ 2 - \frac{q_0^2 - |\vec{q}|^2 - (m_+ + m_-)^2}{4|\vec{k}||\vec{q}|} \ln |A_{-+}| \right\} ,
\end{aligned} \tag{A13}$$

and

$$\text{Im}\tilde{\Sigma}_\pi^R(q_0, \vec{q}) = \pi g_{NN^*\eta}^2 \frac{q_0^2 - |\vec{q}|^2 - (m_+ + m_-)^2}{2} \int \frac{d^3k}{(2\pi)^3} \frac{1}{E_k^+ E_{k-q}^-} \theta(k_{F+} - |\vec{k}|) \delta(q_0 - E_k^+ - E_{k-q}^-)$$

$$\begin{aligned}
& + \pi g_{NN^*\eta}^2 \frac{q_0^2 - |\vec{q}|^2 - (m_+ + m_-)^2}{2} \int \frac{d^3 k}{(2\pi)^3} \frac{1}{E_k^- E_{k-q}^+} \theta(k_{F-} - |\vec{k}|) \delta(q_0 - E_k^- - E_{k-q}^+) \\
& - \pi g_{NN^*\eta}^2 \frac{q_0^2 - |\vec{q}|^2 - (m_+ + m_-)^2}{2} \int \frac{d^3 k}{(2\pi)^3} \frac{1}{E_{k-q}^+ E_k^-} \theta(k_{F-} - |\vec{k}|) \delta(q_0 + E_{k-q}^+ + E_k^-) \\
& - \pi g_{NN^*\eta}^2 \frac{q_0^2 - |\vec{q}|^2 - (m_+ + m_-)^2}{2} \int \frac{d^3 k}{(2\pi)^3} \frac{1}{E_{k-q}^- E_k^+} \theta(k_{F+} - |\vec{k}|) \delta(q_0 + E_{k-q}^- + E_k^+) \\
& - \pi g_{NN^*\eta}^2 \frac{q_0^2 - |\vec{q}|^2 - (m_+ + m_-)^2}{2} \int \frac{d^3 k}{(2\pi)^3} \frac{1}{E_k^+ E_{k-q}^-} \theta(k_{F+} - |\vec{k}|) \delta(q_0 - E_k^+ + E_{k-q}^-) \\
& - \pi g_{NN^*\eta}^2 \frac{q_0^2 - |\vec{q}|^2 - (m_+ + m_-)^2}{2} \int \frac{d^3 k}{(2\pi)^3} \frac{1}{E_k^- E_{k-q}^+} \theta(k_{F-} - |\vec{k}|) \delta(q_0 - E_k^- + E_{k-q}^+) \\
& + \pi g_{NN^*\eta}^2 \frac{q_0^2 - |\vec{q}|^2 - (m_+ + m_-)^2}{2} \int \frac{d^3 k}{(2\pi)^3} \frac{1}{E_{k-q}^+ E_k^-} \theta(k_{F-} - |\vec{k}|) \delta(q_0 - E_{k-q}^+ + E_k^-) \\
& + \pi g_{NN^*\eta}^2 \frac{q_0^2 - |\vec{q}|^2 - (m_+ + m_-)^2}{2} \int \frac{d^3 k}{(2\pi)^3} \frac{1}{E_{k-q}^- E_k^+} \theta(k_{F+} - |\vec{k}|) \delta(q_0 - E_{k-q}^- + E_k^+) . \quad (\text{A14})
\end{aligned}$$

Note that in a limit of $q \rightarrow 0$, $\text{Re}\tilde{\Sigma}_\pi^R(q_0, \vec{q})$ in Eq. (A3) vanishes together with the gap equation in Eq. (29) in the chiral limit $m_\pi \rightarrow 0$. Namely, when we take infinite

sums of self-energies in Fig. (3), pion becomes a massless particle which is consistent with the behavior of a Nambu-Goldstone boson.

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